wonder how the energy can go from the source b to the target f. To understand easily this mechanism we can construct another representation of the topology. It is equivalent to spread the network which is a planar one (it has four faces plus the exterior: its genus is 5). We can associate each vertices to a segment of a circle, and each frontier which is a cord to a radius. A little circle shows the location of the source and a little straight line that indicates the receiver. The figure 7.12 shows this new representation.

On this figure and under this representation, we easily understand that to go from A to D we have to cross the borders $-d$ and h. By the other way the transfer function is hc. All the interactions of the network can be include in a single circle. As the cords considered here are branches, the topology has five faces. But finally, the number of faces is not so interesting to understand how the system works. As there are four segments, the dimension of the manifold is four. The drawn gives also another information. If we want to isolate the source, we can decrease the values of d and h. **The system can be called circular and denoted $\mathcal{S}^{1/2}$** as the interactions can be completely projected on a circle (two circles making a sphere).

Each quarter of the circle is a vertex. But for the moment, this representation is valid only in low frequencies. At least inductances must be added to each vertices. This can include self inductance of borders. As a consequence, this will decrease the quality of the isolation. If we increase again the frequencies, radiated mutual inductance will appear which can add a new direct path from vertex A to vertex C. This new cord can be seen as a new frontier between A and D (imagine that two loops

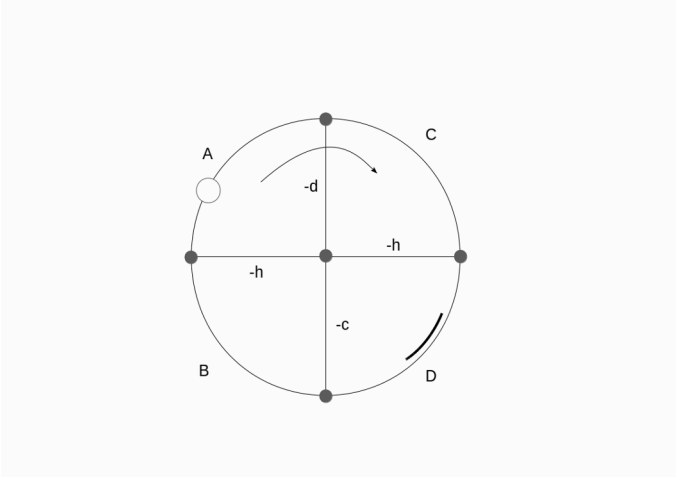

Figure 7.12: Representation in a single circle

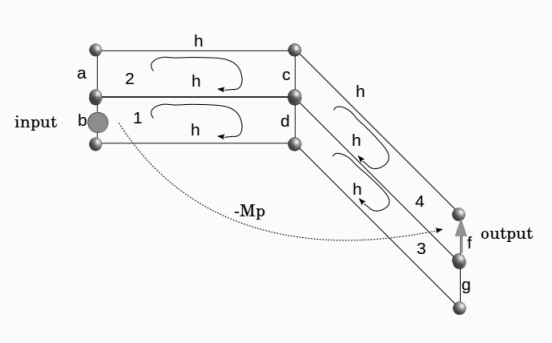

Figure 7.13: Mutual interaction between A and D

in interaction exists at these extremities). Is it possible to add this cord inside the circle?

The figure 7.13 shows this added coupling. As it is a frontier, we may want to add a radius to the previous circle. But it is not possible without making a new vertex incorporated between the others, and no vertices are created using the mutual inductance coupling.

The only solution is to add the new radius on a new half circle with a circulation going from A to D. The figure 7.14 represents this mechanism. The system in this case is $\mathcal{S}^{3/4}$.

The mutual inductance doesn't change the expression of H (equation 7.31) but the metric part $\mathcal{L}$ of the system which be-

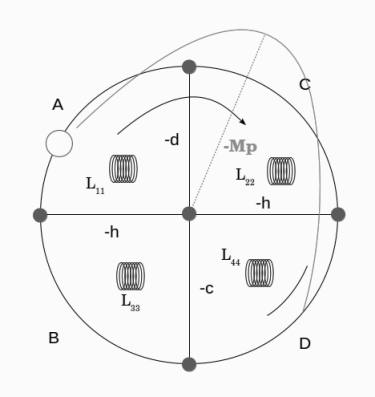

Figure 7.14: Adding a half circle for M

comes:

$$\mathcal{L} = \begin{bmatrix} L_{11} & 0 & 0 & -Mp \\ 0 & L_{22} & 0 & 0 \\ 0 & 0 & L_{33} & 0 \\ -Mp & 0 & 0 & L_{44} \end{bmatrix} \tag{7.32}$$

Seeing this new representation, it is easier to find solutions in order to increase the isolation of a vertex. Clearly in our imagined problem, decreasing the value of d and h is no more sufficient, it is also necessary to suppress the M interaction. Now finding the influence of each path is a question of computation, using the distance definition given equation 7.26.

A first conclusion on the systems analysis

To represent the interactions inside a system is a complicated task. Starting from the real system, we describe it as a collection of subpart or sub-manifolds and by a process of direct sum and adding of interactions, we construct the complete manifold associated with the whole system. From this definition, we make a projection of the lagrangian into a graph made of vertices and edges as meshes and cords. In a second step we transform this representation of the manifold in circles defining $\mathcal{S}$ systems or manifolds. Then, solutions can be studied to decrease some paths between emitters and receivers. The distance between them is defined in volts, as demonstrated through the second geometrization process.

Increasing the frequencies and $\mathcal{S}^n$ systems analysis

We continue to discuss with our first case. If the frequencies increase, the extremities of each line will be separate and no longer linked by a non propagative model of wire. Each line will be represented by a branin. The dimension of the manifold increases and include eight meshes. Between the extremities of the line we have a cord and between lines, another cord defined by Vabre's relations[3]. But we don't care here of Vabre's formula. The figure 7.15 shows the transformation between the graph and its projection on a circle as we do previously.

[3]Alain Reineix, Olivier Maurice. Progrès récents dans la modélisation CEM de câblages électriques de systèmes complexes. CCT-CEM, Mar 2017, Toulouse, France. hal-01495620

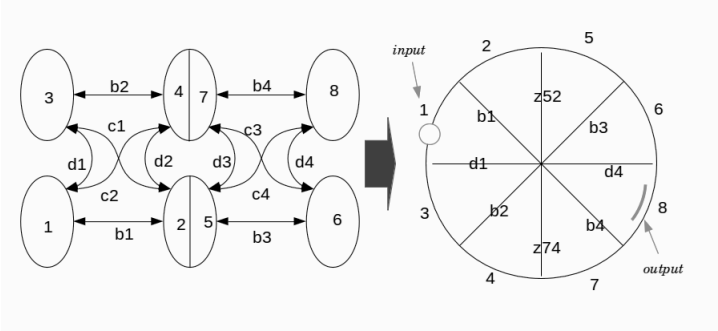

Figure 7.15: High frequency projection on the circle

We have construct the circle starting from the major path between the input and the output. This leads to the circle as it is presented on the figure 7.15. But other paths exist, the paths using the far-end cross talk between the lines. And if we trace them, this gives the first drawn figure 7.16. We understand that we cannot trace these interactions without crossing them if we keep them either on the same circle, or even on a new half circle. But using a new complete circle, it is possible to create paths without any cutting the paths. A first circle is drawn taking into account the interactions $c2$ and $c4$ and a second circle is drawn taking into account the interactions $c1$ and $c3$. The figure 7.17 shows the first circle. The whole set of three circles describes completely the system. Note that all half circles are completed by some vertices in order to wear the total interactions from

the source 1 to the target 8. Having three circles in final, the system is $\mathcal{S}^{3/2}$. Two circles giving a sphere ($\mathcal{S}^1$) and with one circle more, this leads to $\mathcal{S}^{3/2}$.

The higher the order of $\mathcal{S}$, the higher the difficulty to isolate the target from the source. For finding a solution in order to avoid disturbances on the function 8 coming from emissions of 1, it's necessary to reduce the problem if possible. First operation is to apply the "*weak coupling approximation*". Under the "weak coupling approximation" (WCA) we accept the idea that the reaction of the receiver of energy doesn't impact the behavior of the emitter. In other word we may say that the coupling function has an impedance operator that is not changed depending on the receiver. This is false when near field coupling arrives in the process but globally true elsewhere. This is a consequence of the diffusion process. Often the emitter communicates its energy to many paths, and the receivers make the same. Of fact the part of energy that comes back to the emitter, coming from one receiver is neglectable. In near field process, both emitter and receiver become a unique object inside which energy is exchanged. The diffusion in that case is out of purpose and both emitter and receiver influence the whole system realized starting from their common participation.

How is it possible to verify the WCA? For an emitter u and a receiver J (natural written opposite to the covariant one) linked through a coupling function y, if the coupling is a WCA one:

$$J(u) = J(u) \pm \epsilon, \forall J \Rightarrow \frac{\Delta J(u)}{\Delta J} < \epsilon = f\left(y^2\right) \tag{7.33}$$

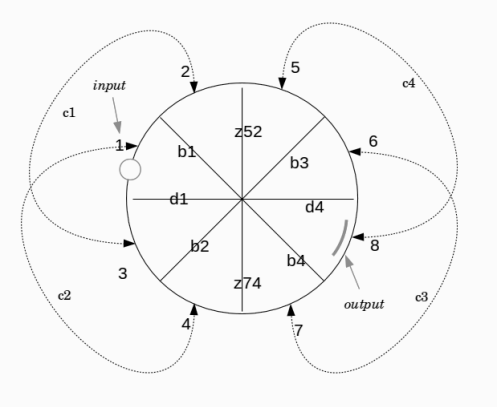

Figure 7.16: Adding cords from far end cross talk

ϵ being a residual change in the current of the emitter.

It is important to understand that this doesn't mean that a current on the receiver $J(u)$ cannot change the current of the emitter J. Because, at the inverse, it can exist a WCA from u seen as an emitter to J seen as a receiver. But it means that the back energy resent to the emitter and coming from itself is neglectable. This can be true even for non symmetrical couplings.

If we have $J = yu$. If z is the impedance operator of J, we have also $v = J/z$. Now we can write $\Delta J(u) = y'v$, so by

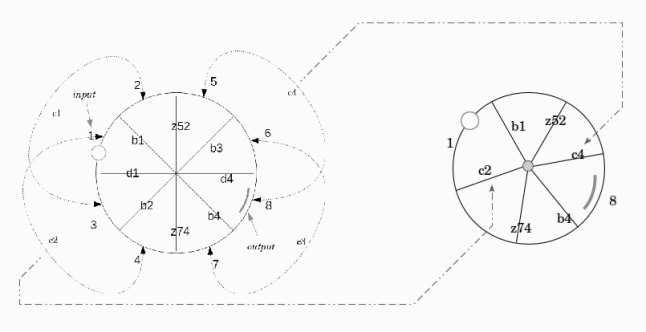

Figure 7.17: Two circles more for other paths

replacement this gives $J(u) = y'J/z$ and finally:

$$\Delta J(u) = \frac{1}{z} y' y u$$

Our criterion becomes:

$$\frac{1}{z} y' y u < \epsilon \tag{7.34}$$

Note that the difficulty comes from the fact that the criterion depends on the impedance operator of the receiver. A way to measure this effect is to measure the impedance seen from the input of the emitter. If the current is influenced by a potential receiver, the impedance will change also, because it means that the generator see another operator on the input. And if the impedance operator is modified by the proximity of another

one that belongs to the receiver, we understand easily that this modification depends on the impedance operator of the receiver.

There is a domain where this problem is clearly identified: the antennas. As long as the environment doesn't influence an antenna, its radiation impedance remains unchanged. If an object becomes to be too much near, the radiation impedance changes and the new antenna is constitued by including both the original antenna and the object.

Under the WCA we can triangularized our matrix H. In low frequencies H becomes:

$$H = \begin{bmatrix} 2h+b+d & 0 & 0 & 0 \\ -h & 2h+a+c & 0 & 0 \\ -d & 0 & 2h+d+g & 0 \\ 0 & -c & -h & 2h+c+f \end{bmatrix} \tag{7.35}$$

and the same for $\mathcal{L}$:

$$\mathcal{L} = \begin{bmatrix} L_{11} & 0 & 0 & 0 \\ 0 & L_{22} & 0 & 0 \\ 0 & 0 & L_{33} & 0 \\ -Mp & 0 & 0 & L_{44} \end{bmatrix} \tag{7.36}$$

With the assumption of WCA, each path becomes easy to establish. Starting from a current on A (covariant approach) on

vertex B we can write:

$$e_B = -hJ^A$$

passing through a first border. Now $J^B = e_B/(2h + a + c)$ and $e_D = -cJ^B$. So:

$$J^D = \frac{hc}{(2h + c + f)(2h + a + c)} J^A$$

This first transfer function ft associates with this first path is defined by:

$$ft = 20log\left(\frac{J^D}{J^A}\right) \tag{7.37}$$

It remains to compute the other paths... The transfer function results from the product of the border functions on the numerator and the product of the impedance operator on the denominator (with WCA and linear assumptions). Bode's diagram is easy to trace as solutions to reduce the impact of various paths.

[**First reduction process**] A question may be: how to do with strong interactions? There is only one possible response: **a set of branches leading to a network where strong interactions exist must be considered as a single vertex**. The associate manifold is said to be unseparable. On the circle projection, this can be include by making abstraction of the detail inside a quarter. The attention must be focused on the borders for which the WCA can be applied. This constitutes the **first reduction process**.

[**Second reduction process**] The second reduction process consists in decreasing as far as possible the order of $\mathcal{S}$. Our previous system being $\mathcal{S}^{3/2}$, an ideal situation may be to reach $\mathcal{S}^{1/4}$ (**segregation principle**)! To do that, it seems simplest to begin by the paths of lower orders, i.e. where the number of borders is the lower. There is a thin relation between the number of borders and the neighborhood. This conducts to define a neighborhood distance:

Definition:

The neighborhood distance $\hat{d}$ is the number of borders to cross going from an emitter to a receiver.

The strategy used to decreased the coupling can be to increase the geometrical distance: i.e. playing on $\mathcal{L}$, or playing on the filters and impedances defined in the operators of H. We find the projections on the diagram 7.5 with displacements along the prevention axis or the protection axis.

Let's take an example. We consider a circuit including two meshes separated by a capacitive filter. A part of this circuit is coupled with a pigtail of a receiver equipped with a shielded cable. Another path is between the same part emitting of the first circuit and the external domain of the receiver. Then through the transfer impedance of the shield, it creates an EM force in the internal domain. The vertex graph of the problem is presented figure 7.18 where we show voluntarily the detail of the two meshes for the first circuit.

The representation of this graph into circles is given figure 7.19. The first reduction was applied, reducing the two meshes of the first circuit into a single part of the circle. Two paths are available to go from b to the internal domain D_i. The order

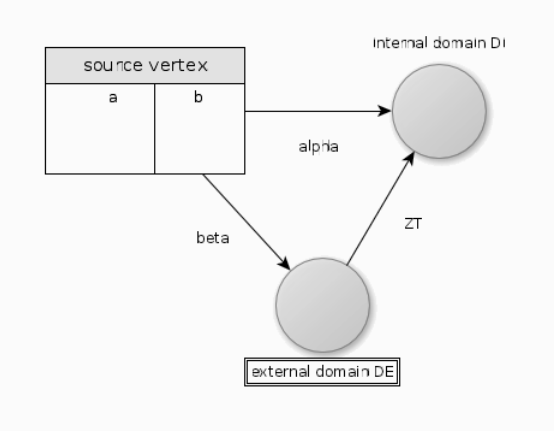

Figure 7.18: A S1/2 system

of the system is $\mathcal{S}^{1/2}$. The first neighborhood of the target is the coupling through the pigtail α. By suppressing the coupling through the pigtail, we suppress one path and remains only the coupling through the transfer impedance of the cable. The system becomes $\mathcal{S}^{1/4}$, last dimension before $\mathcal{S}^0$. The neighborhood distance is $\hat{d} = 2$. The transfer function is given by:

$$J^{D_i} = -\frac{1}{Cp}\frac{\beta Z_T}{bD_E D_i} J^a$$

that shows clearly that decreasing Z_T will increase the protection of J^{D_i}.

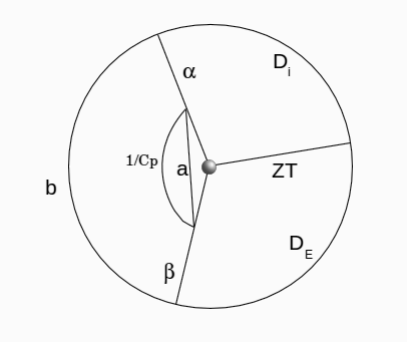

Figure 7.19: Circle representation

In this result, increasing Z_T increase the protection of the receiver. The other parameter is the coupling cœfficient β which has the dimension of a mutual inductance. If the geometrical distance between the emitter and the shielded cable increases, this will decrease the EM force induced on the external domain, and, as a consequence, decrease the level induced on the target. For an identical level of protection, we can maintain the product βZ_T constant, or near a given value. β is part of the metric $\mathcal{L}$ while the transfer impedance function is part of the structure H. We can construct a PP diagram (see figure 7.5) to trace the limit curve of safety. Depending on the number of layers for the shield and depending on the distance we obtain three states that give the limit of the PP curve. Figure 7.20 illustrates the process.

Now if for some reason of system layout we need to reduce the distance between the emitter and the receiver, this operation can locate the point of working under the limit, like shown figure 7.21. In that case there are two solutions:

- looking for the nearest point on the limit, which will imply a compromise on both distance and protection;
- to keep the distance desired and looking for the protection involved by this change: here a three layers shielded cable.

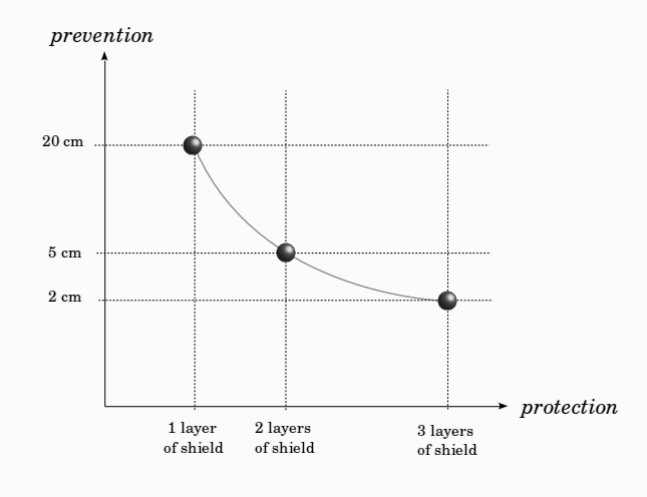

Figure 7.20: Limit curve in the PP diagram

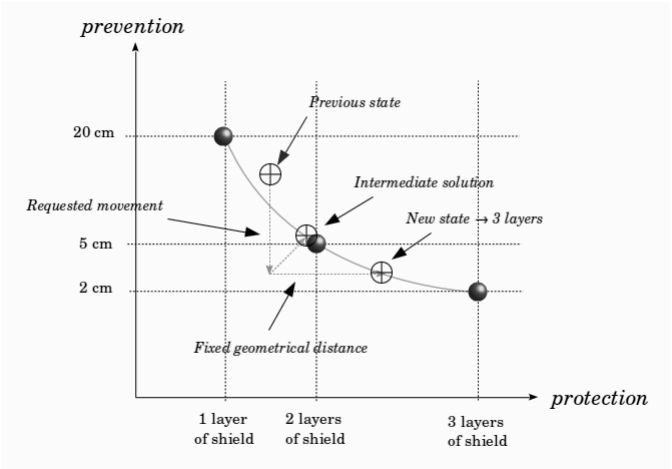

Figure 7.21: Finding solution after a new layout

For analyzing the evolution of the risk of failure in a system consists in following the trajectory of its working point in the PP diagram. Each changing in this film accompanies the movement of the system depending on the phases of its mission and the evolution of its environment. The threshold U_s is the input data from the data sheet giving the level of sensibility for the device. It can be computed for out-band behavior using research works and other sources of information. This value has a given uncertainty and lack of knowledge. For this reason a margin must be taken to decrease this value and to guarantee that the electronic won't be disturbed. Now, around this new threshold, the whole uncertainty including those of the structure, components, locations, etc., should not reach the previous initial value obtained from the data sheet. Figure 7.22 illustrates this mechanism.

To remain under the level fixed, gain in the protection cannot go linearly from one value to another. We see that adding a braid on a cable will give one step 20 dB at least of isolation. That's often the case whatever the constraints and this shows the mechanism in a system conception and the limit in an optimization process.

[What can be done when emitter and receiver are in strong coupling interaction?] Same process must be followed but the engineer should remember that under strong coupling assumption, both emitter and receiver will be impacted by any change in the coupling function.

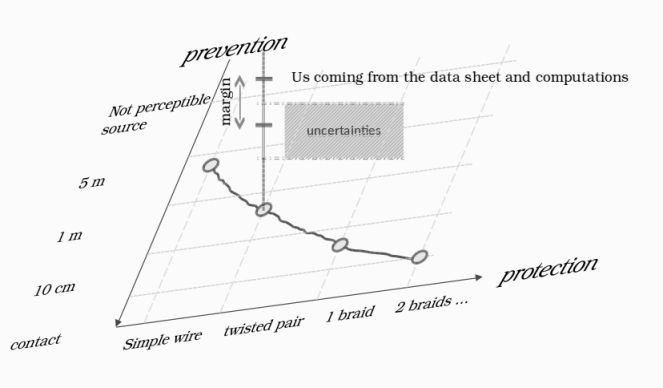

Figure 7.22: Uncertainty around the threshold

7.5.3 Margin management

Once the border determined, the cycle from the emission limits defined to the immunity limits requirements can be made. For a threshold V_s of a component, the choice of architecture leads to a maximum of emissions for all sources M_E. On this level we take a margin to define the immunity limit $I_L = M_E + margin$. This constraint generates a level across the component V_g. Logically, $V_g = V_s + margin$. If no disturbances is observed during the test, it confirms that the margin is reached. The figure 7.23 explains the process.

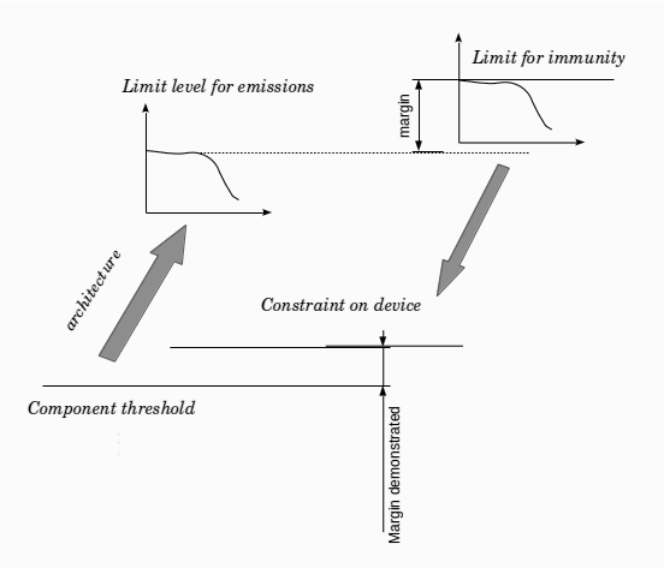

Figure 7.23: Margin process

7.5.4 System life

During its life or its mission, a system changes in its characteristics and can encountered more or less external emitters.

When the system evolves, the story can be drawn on a musical partition. At each step, the metric and the hamiltonian of the system can change like the external sources radiating into the direction of the system. But the relative location of the system and of the sources can change also.

The source vector (using a nodes pair vector) evolves depending on the system movement or evolution. The components of the vector source change depending on time, and the way they are seen by the system is also changing with time. This changes can be taken into account using a matrix that I have called a gamma matrix.

Figure 7.24 shows how the system life can be seen compared to a music sheet.

Once the system chains are conceived, the system can appear like a robot, able to make various actions in response to commands. Depending on its "autonomy", this capacity goes more or less far. For example we can imagine all the versions we have between two robots, a first one being a classical car. I use it to go from Paris to Rouen. It helps me to make this travel with different secure functions, but I have to drive this car anyway! And a second robot which ask me the destination, and make alone all the rest to transport me to this destination.

Under this kind of system, what evolves principally during the mission is the environment. In parallel to the previous music sheet, we can add another one to see how the environment

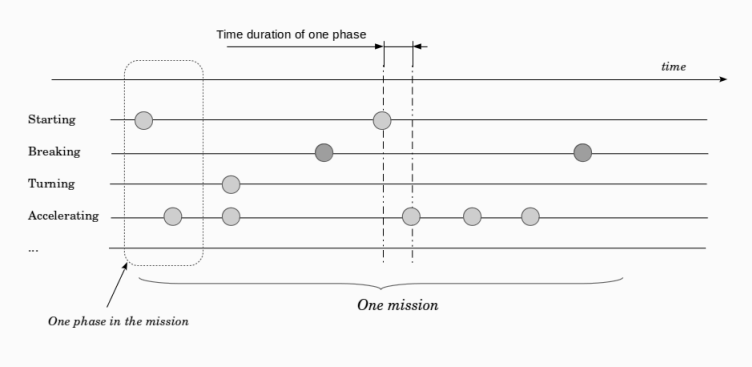

Figure 7.24: System life, mission and phases

evolves during the same mission. Each line takes in charge one source that belongs to the environment. So the problem consists in having a mathematical technique that changes the source covector following its real changing in the mission. Note that the synchronizations (C_n) between the existence of a source in the environment and the electronic state of the system obliges to study the problem through a stochastic process. Each electronic state as phase of mission can be identified with color in correspondance with their critical role in the mission. The same for the constraints that can be more or less dangerous. More, the time duration of each event can be drawn on the music sheet. But a difficulty is that for example, between the time duration of a breaking phase for a vehicle and the duration of lightning

field, the difference of scales is enormous. Figure 7.25 illustrates this approach.

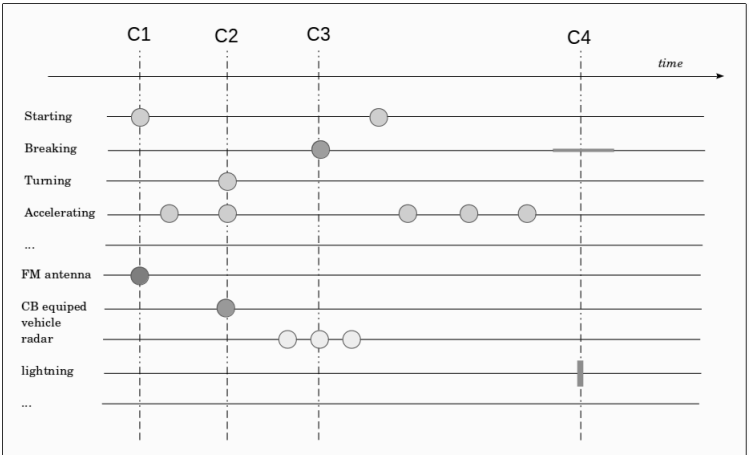

Figure 7.25: System life and environment

The system must be cut in subparts to be studied. It is impossible for the human mind to encompass the whole system. Doctors need to understand each subparts of a human body to make a diagnostic answering to a list of symptoms. The system is made of chains. One chain to distribute the energy. One chain to distribute the intelligence (numeric chain and microprocessors). One chain to manage the perception. One chain to manage the actions (motors, etc.). And so on... Each chain is made of signals exchanged between electronic equipments involved in

this chain. Each equipment is made of electronic components. All of this mechanisms are supported in a mechanical structure or in various mechanical structures. Strategies of exoskeleton or endoskeleton can be used. It's the same for the conception of the system. Two approaches can be followed: bottom-up and up-down. Sometimes, both approaches are used. There are no rules. In final, chains must be defined and incorporated in a chosen structure. What we may say is that the more the system incorporates electronic, the more the bottom-up approach should be used. This because the electronic can adapt the structure to answer to various sollicitations, even mechanical ones. In that case, the mechanical structure can be defined only once the electronic functions are defined themselves.

In any cases, the objectives of the system is to make some missions, to answer for requirements of the customer.

Each chain q is associated with an impedance operator $\overset{q}{\zeta}$. Each of this operator is created using a direct sum of the operators of the equipments $\overset{e}{\zeta}$. Each operator of equipment is constructed from the direct sum of the components operators: $\overset{e}{\zeta} = \oplus_c \overset{c}{\zeta}$. But rather than speaking of operators, we should speak of manifolds. And finally, the system S appears as a direct sum of manifolds:

$$S = \oplus_q \left[\oplus_e \left(\oplus_c \overset{c}{\zeta} \right) \right] \tag{7.38}$$

and $\oplus_c \overset{c}{\zeta} = \overset{e}{\zeta}$, and $\overset{q}{\zeta} = \oplus_e \overset{e}{\zeta}$. But we known that these direct sums must be completed by coupling operators, including the

metric $\mathcal{L}$.

This has a deep physical meaning. Constructing a system supposes that you group the system in the same space, under the same metric. If not, it becomes impossible to join its pieces or to establish a dialog between its microprocessors. To connect both metrics of the separated systems, the influence of each metric must be transferred to the other. That's the purpose of the mutual inductance. Once more we may discuss of the weak coupling assumption meaning that a system part is weakly coupled to the other part of the system. This means that the coupling acts only one way without back effect on the emitter. But at the system level, this doesn't mean that the added part has no influence on the system. The assumption of weak coupling can concern one physic, and not the whole system mechanisms. For example, adding a microprocessor can change the energy consumption at the margin but decide of the whole system behavior.

7.5.5 Sources and gamma matrices

The sources of the environment can appear many times in different locations, or be seen by various parts of the system because of its movements, or again appear only one time in a particular moment. Knowing Markov's processes, a similar matrix can help to represent this kind of changing in the environment. We just need to replace the source covector e by a modified source covector γe depending of some rhythm that helps to follow the system life.

When we need to study a system, a permanent problem ap-

pears: is it possible to model the system in the frequency domain or is it possible to model it only in the time domain? Why this question? Because they are physical phenomenons that can be easily modeled in the frequency domain, while others can be easily modeled in the time domain.

Skin effect, modes in a cavity, radiations are some of the physical phenomenons that are easier to model in the frequency domain. At the opposite, non linear behaviors like diodes are easier to model in the time domain. In fact, we can consider two facts very importants in this problem:

1. non linear behaviors often depends on low frequency signals and polarisation signals;

2. the first difference between a periodic temporal form and a single temporal form is that in the first case, the spectrum is made of raies (the amplitude is also modified).

A graph in low frequencies decides of the state of the non linear components. The high frequency components are modified depending on these states. The low frequency spectrum is determined making the signal periodical even if this is not the case in the real world. During the computation, a temporal loop makes the non linearities changing and inside this temporal loop, a frequency loop takes in charge the computation of the high frequency signals. This idea uses two important results:

1. when a signal is called "little signal" it has laws defined on only one domain;

2. the signals that makes the polarization of the nonlinear components evolves slowly in a way that allows to neglect the majority of the capacitors and inductances of low values. This simplifies the circuit and the expression of its impedance operator. This is always interesting in the temporal domain.

In a first step we make the signal periodic: $e(t+T) = e(t)$. This allows to find accurately the low frequency spectrum. Each part of $e(t)$ can be retrieved making a moving average computation on the signal. If f_h is the typical central frequency of the high frequency part of the signal and f_b its low frequency envelop, if $f_b << f_h$, the average of the signal on a multiple of the duration $1/f_h$ tends to suppress the high frequency signal without impacting the low frequency one.

Once the computation is made, the amplitude found on each high frequency component decides of its amplitude in the temporal sum. To obtain the time domain signal, it's just necessary to add these various sinusoidal components to the low frequency envelop. This kind of strategy is not so easy to write. After having verified that the high frequency signal cannot develop the level responsible for the changes in the domains, we determine two circuits:

1. the low frequency circuit solved in the time domain with the influence of the non linear components. Its source is $q(t)$;

2. the high frequency circuit solved in the harmonic domain of sources $L_h(f)$.

at each time step, we solve the low frequency circuit using time domain finite difference or Newton's method. For each of these steps, we solve the whole frequency domain for the high frequency circuit. The amplitude of the harmonics are memorized then added to the complete solution:

$$s(t) = \zeta[q(t) + \sum_f L_h(f)Cos(2\pi ft)]$$

One advantage of this decomposition is to take into account changes in the high frequency part, mostly if the high frequency signal comes from a different source from the low frequency one.

We write the signal $u(t)$ with the sum $q(t) + h(t)$ where $q(t)$ is the slowly varying signal with the spectrum $L_q(f)$ and $h(t)$ the high frequency signal with the spectrum $L_h(f)$.

A symbol with a hat points out the spectrum of the signal identified by the symbol $(e(t) \to \hat{e}(f))$.

If we are in small signal conditions, the change in this voltage or in the corresponding current should not influence the impedance operator. The small signal condition can be written:

$$\forall \hat{e}_\nu, \hat{i}^\alpha \in L_h(f), \frac{\partial \zeta_{\nu\sigma}}{\partial i^\alpha} = 0 \tag{7.39}$$

This can be said with the condition: $\Gamma_{\nu\sigma,\alpha} = 0$. Once more the curvature plays a very important role. We see that as often, working in a flat space simplifies enormously the analysis.

We imagine a signal defined by:

$$u(t) = \alpha Cos\left(2\pi \frac{t}{T_B}\right) + ACos\left(2\pi \frac{t}{T_0}\right) = v(t) + b(t)$$

We wonder how we may treat both signals high and low frequencies which means to be able to identify them in a first step? We make the assumption that the non linear behavior has a cutoff frequency f_c and period T_c. We have $T_B >> T_c$ and $T_0 << T_c$. We want to compute a floating average defined by:

$$\langle u \rangle (t) = \frac{1}{T_0} \int_t^{t+T_0} dt u(t) \tag{7.40}$$

This leads to:

$$\langle u \rangle (t) = \frac{\alpha}{T_0} \int_t^{t+T_0} dt Cos\left(2\pi \frac{t}{T_B}\right) + \frac{A}{T_0} \int_t^{t+T_0} dt Cos\left(2\pi \frac{t}{T_0}\right)$$

We obtain:

$$\frac{\alpha T_B}{2\pi T_O}\left[Sin\left(2\pi \frac{t+T_0}{T_B}\right) - Sin\left(2\pi \frac{t}{T_B}\right)\right] + \dots$$

$$\dots + \frac{AT_0}{2\pi T_0}\left[Sin\left(2\pi \frac{t+T_0}{T_0}\right) - Sin\left(2\pi \frac{t}{T_0}\right)\right]$$

We look at the second term:

$$\dots = \frac{A}{2\pi}\left[\left\{Sin\left(2\pi \frac{t}{T_0}\right) Cos 2\pi + Cos\left(2\pi \frac{t}{T_0}\right) Sin 2\pi\right\} - Sin\left(2\pi \frac{t}{T_0}\right)\right]$$

$$\dots = 0$$

The first term gives:

$$\dots = \frac{\alpha T_B}{2\pi T_0}\left[\left\{Sin\left(2\pi \frac{t}{T_B}\right) Cos\left(2\pi \frac{T_0}{T_B}\right) + Cos\left(2\pi \frac{t}{T_B}\right) Sin\left(2\pi \frac{T_0}{T_B}\right)\right\}\right.$$

$$\left. \dots - Sin\left(2\pi \frac{t}{T_B}\right)\right]$$

But:

$$\frac{T_0}{T_B} = \epsilon \to 0$$

so:

$$\ldots = \frac{\alpha T_B}{2\pi T_0}\left[Cos\left(2\pi\frac{t}{T_0}\right)Sin\left(2\pi\frac{T_0}{T_B}\right)\right]$$

and:

$$Sin\left(2\pi\frac{T_0}{T_B}\right) \to 2\pi\frac{T_0}{T_B}$$

Finally:

$$\langle u\rangle(t) = \alpha Cos\left(2\pi\frac{t}{T_B}\right) = v(t)$$

the high frequency and small signal $b(t)$ has disappeared. The envelop of the composite signal can be extracted for a known cutoff frequency of non linear behavior.

The previous demonstration can be extended to any kind of envelop, not only the sinusoidal ones. This can be shown just defining:

$$v(t) = \sum_q \alpha_q Cos\left(2\pi\frac{t}{T_q}\right)$$

with $T_q >> T_0$, $\forall q$.

In order to facilitate the distinction between both kind of signal, it is sometimes interesting to use also both time and Laplace's operator in a same equation. The envelope is treated using the time and the small signal component using Laplace's operator. This appears naturally in the program where inside the frequency loop there are equations depending on time and

also depending on the pulsation. The time part of the equation changes the polarization and the impedance operator, while the small signal part evolves depending on the impedance sets by the envelope. The impedance changes because parameters like voltages or currents set domains that determine the expression of the impedance operator. This can be somewhere written: $\hat{e}_a = \zeta_{ab}\left[v(t)\right]\hat{i}^b$.

We imagine a fixed polarisation source $p_k(t)$ and a system passing in front of this source. If the system has N inputs, the source can illuminate various inputs of the system during time. If the covector e at the origine is given by (for three inputs in the system):

$$e = \begin{bmatrix} p_1(t) \\ 0 \\ 0 \end{bmatrix} \tag{7.41}$$

If, after sometime, the movement of the system exposes the second input to the source, the matrix γ defined by:

$$\gamma = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7.42}$$

translates this process.

What is particularly interesting in this technique is that the matrix γ can have for components the probabilities that such a source illuminates a given input of the system. More often, these probabilities are bayesian probabilities. We write the probability that the source takes a value, knowing that the configuration

of the system is also a first one, etc.:

$$\gamma_{(k)1} = P\left(e_{(k)} = e_1 \middle| \, x^k, \zeta, \ldots\right)$$

The commands e_k drive the behavior of the system and the external commands depends on stochastic processes. A connectivity make links between a set of sources determined by a γ matrix that selects the concerned sources and the covector source applied to the system. We start from a known set of sources $\{e_a\}$. We identify in this set the sources concern by the system life in a given moment: $\gamma_b^a \{e_a\}$. Then a connectivity creates the adequate source covector for the studied manifold:

$$u_k = C_k^b \gamma_b^a \{e_a\}$$

The use of gamma matrices can be very fun for modeling of propagating waves. Let's take an example. We imagine a system with two vertices. The source vector is defined by:

$$U_\alpha = \begin{bmatrix} e_1 \\ 0 \end{bmatrix} \tag{7.43}$$

The propagation of the waves between these two vertices is defined by the gamma matrix:

$$\gamma_\alpha^\alpha = \begin{bmatrix} 0 & -G \\ -G & 0 \end{bmatrix} \tag{7.44}$$

G is a Green's function. If we compute $U_\alpha = \gamma_\alpha^\alpha U_\alpha$, this gives:

$$U_\alpha = \begin{bmatrix} 0 \\ -Ge_1 \end{bmatrix} \tag{7.45}$$

$U_\alpha = \gamma^\alpha_\alpha \gamma^\alpha_\alpha U_\alpha$ gives:

$$U_\alpha = \begin{bmatrix} G^2 e_1 \\ 0 \end{bmatrix} \tag{7.46}$$

$U_\alpha = \gamma^\alpha_\alpha \gamma^\alpha_\alpha \gamma^\alpha_\alpha U_\alpha$:

$$U_\alpha = \begin{bmatrix} 0 \\ -G^3 e_1 \end{bmatrix} \tag{7.47}$$

$U_\alpha = \gamma^\alpha_\alpha \gamma^\alpha_\alpha \gamma^\alpha_\alpha \gamma^\alpha_\alpha U_\alpha$:

$$U_\alpha = \begin{bmatrix} G^4 e_1 \\ 0 \end{bmatrix} \tag{7.48}$$

etc. This gamma matrix models the ping pong of a wave exchanged between the two vertices. If $G = 1$, it is an oscillator.

Let's take another example. We consider a system of three vertices linked by two channel of communication. The part of energy transmitted on vertex 2 is given by the cœfficient $y_{21} = 1 + \sigma_{12}$, where σ_{12} is the reflexion cœfficient at the border on vertex 2. Another part of the energy is reflected to vertex 1, etc. These cœfficients are weighted by the delays of propagation between each vertex. Finally the gamma matrix is defined by:

$$\gamma^\alpha_\alpha = \begin{bmatrix} 0 & \sigma_{12} e^{-\tau_1 p} & 0 \\ y_{21} e^{-\tau_1 p} & 0 & \sigma_{23} e^{-\tau_2 p} \\ 0 & y_{32} e^{-\tau_2 p} & 0 \end{bmatrix} \tag{7.49}$$

τ_x are the delays of each channel.

Starting from the source vector:

$$U_\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{7.50}$$

we obtain:

$$\gamma_\alpha^\alpha U_\alpha = \begin{bmatrix} 0 \\ y_{21} e^{-\tau_1 p} \\ 0 \end{bmatrix} \tag{7.51}$$

$$\gamma_\alpha^\alpha \gamma_\alpha^\alpha U_\alpha = \begin{bmatrix} \sigma_{12} y_{21} e^{-2\tau_1 p} \\ 0 \\ y_{32} y_{21} e^{-(\tau_2+\tau_1)p} \end{bmatrix} \tag{7.52}$$

etc.

The evolution of the source vector shows how the waves propagates in the system for a source in vertex 1.

7.5.6 Time and frequencies

It is clear that the harmonic domain allows to compute easier complicated phenomenon like the skin effect. Its big disadvantage concerns the non linear behaviors. If we consider a trapezoidal form of rise and fall time t_s and duration t_D, its Laplace's transform is:

$$s(p) = \left(\frac{1 - e^{-t_s p}}{t_s p^2} - \frac{1 - e^{-t_s p} e^{-t_D p}}{t_s p^2} \right) \tag{7.53}$$

We can sometimes approximate a function $u(t)$ as a sum of these kind of signal:

$$u(t) = \sum_\alpha A_\alpha s(p) e^{-\tau_\alpha p} = \sum_\alpha v_\alpha(t) \tag{7.54}$$